Progress on PhD Research

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Comparing Classical Kinematics and Special Relativity using Definitional Equivalence

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Different kinds of equivalence

Logical Equivalence

$\Downarrow \not\Uparrow$

Definitional Equivalence

$\Downarrow \not\Uparrow$

Conceptual Equivalence (Morita Equivalence, definitional equivalence with different sorts)

$\Downarrow \not\Uparrow$

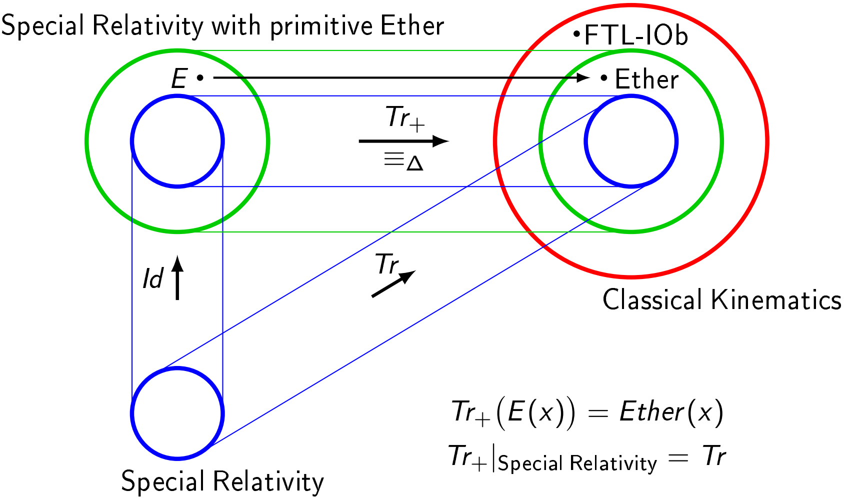
Categorical Equivalence

References:

"*Morita Equivalence*" by Thomas William Barrett and Hans Halvorson

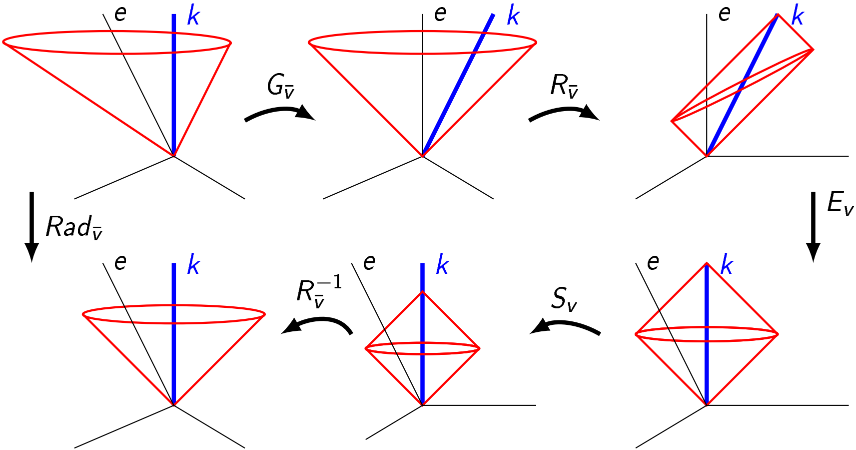
"*Mutual definability does not imply definitional equivalence, a simple example*" by Andréka, H., Madarász, J. X., and Németi, I. in Mathematical Logic Quarterly, 2005.

Recapitulation of previous presentations



$$Tr_{+}(E(x)) = Ether(x)$$

$$Tr_{+}|_{\text{Special Relativity}} = Tr$$



$$Rad_{\bar{v}} = R_{\bar{v}}^{-1} \circ S_v \circ E_v \circ R_{\bar{v}} \circ G_{\bar{v}}$$

$$\mathsf{AxSelf_{SR}} : \ (\forall k \in IOb)(\forall \bar{y} \in Q^4)\left[W(k,k,\bar{y}) \iff y_1 = y_2 = y_3 = 0\right]$$

$$Tr(\mathsf{AxSelf_{SR}}) \equiv$$

$$\forall k \Big(IOb(k) \wedge (\forall e \in Ether)\big(speed_e(k) < \mathfrak{c}_\mathfrak{e}\big)$$
$$\implies (\forall \bar{y} Q^4)(\forall e \in Ether)\left[W\left(k,k,Rad^{-1}_{\bar{v}_k(e)}(\bar{y})\right) \iff y_1 = y_2 = y_3 = 0\right]\Big)$$

Ether Observer Independent formulas

$$\mathsf{AxSelf_{SR}} : \ (\forall k \in IOb)(\forall \bar{y} \in Q^4)\left[W(k,k,\bar{y}) \iff y_1 = y_2 = y_3 = 0\right]$$

$$Tr(\mathsf{AxSelf_{SR}}) \equiv$$

$$\forall k \Big(IOb(k) \wedge (\forall e \in Ether)\big(speed_e(k) < \mathfrak{c}_\mathfrak{e}\big)$$
$$\implies (\forall \bar{y} Q^4)\big(\forall e \in Ether\big)\left[W\left(k,k,Rad^{-1}_{\bar{v}_k(e)}(\bar{y})\right) \iff y_1 = y_2 = y_3 = 0\right]\Big)$$

$$\equiv$$

$$(\forall k \in IOb)(\forall e \in Ether)\Big(speed_e(k) < \mathfrak{c}_\mathfrak{e}$$
$$\implies (\forall \bar{y} Q^4)\left[W\left(k,k,Rad^{-1}_{\bar{v}_k(e)}(\bar{y})\right) \iff y_1 = y_2 = y_3 = 0\right]\Big)$$

$E(x)$: x is even, $O(x)$: x is odd

$$(\forall x)[E(x) \vee O(x)]$$
$$\not\Updownarrow$$
$$(\forall x)[E(x)] \vee (\forall x)[O(x)]$$

$$(\forall x)[E(5) \vee O(7)]$$
$$\Updownarrow$$
$$(\forall x)[E(5)] \vee (\forall x)[O(7)]$$

Assuming **ClassicalKin$_{Full}$**, all ether observers are stationary with respect to each other, and hence they agree on the speed of light.

$$EOI_b^{k_1,\ldots,k_n}[\varphi] \overset{def}{\iff}$$
$$\textit{ClassicalKin}_{\textit{Full}} \models (\forall k_1,\ldots,k_n \in IOb)(\forall e,e' \in Ether)$$
$$[\varphi(e/b) \iff \varphi(e'/b)].$$

where $\varphi(k/b)$ means that b gets replaced by k in all free occurences of b in formula φ.

The following rules can be used to show the ether independence of complex formulas:

1. From $EOI_b^{k_1,\dots,k_n}[\varphi]$ follows $EOI_b^{k_1,\dots,k_n}[\neg\varphi]$.
2. If $*$ is a logical connective, then from $EOI_b^{k_1,\dots,k_n}[\varphi]$ and $EOI_b^{h_1,\dots,h_m}[\psi]$ follows $EOI_b^{k_1,\dots,k_n,h_1,\dots,h_m}[\varphi * \psi]$.
3. From $EOI_b^{k_1,\dots,k_n}[\varphi]$ follows $EOI_b^{k_1,\dots,k_n}[(\exists x \in Q)(\varphi)]$ and $EOI_b^{k_1,\dots,k_n}[(\exists \in hB)(\varphi)]$.
4. From $EOI_b^{k_1,\dots,k_n}[\varphi]$ follows $EOI_b^{k_1,\dots,k_n}[(\forall x \in Q)(\varphi)]$ and $EOI_b^{k_1,\dots,k_n}[(\forall h \in B)(\varphi)]$.

Let k be a body variable let and $\bar{\alpha}$ and $\bar{\beta}$ quantity terms.
Then for any body variable b we have:

1. $EOI_b^k[Rad^{-1}_{\bar{v}_k(b)}(\bar{\alpha}) = \bar{\beta}]$.
2. $EOI_b^k[W(k, h, Rad^{-1}_{\bar{v}_k(b)}(\bar{\alpha}))]$.

Let us assume $EOI_e^{k_1,\ldots,k_n}[\varphi]$ and $EOI_e^{h_1,\ldots,h_m}[\psi]$.
For every logical connective $*$,

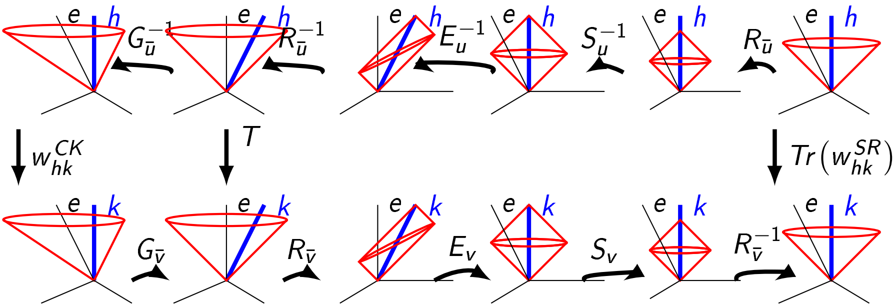
$$(\forall e \in Ether)(\varphi) * (\forall e \in Ether)(\psi)$$

is equivalent to

$$(\forall e \in Ether)(\varphi * \psi)$$

provided that $k_1, \ldots, k_n, h_1, \ldots, h_m$ are inertial observers.

A cannon to shoot mosquitos

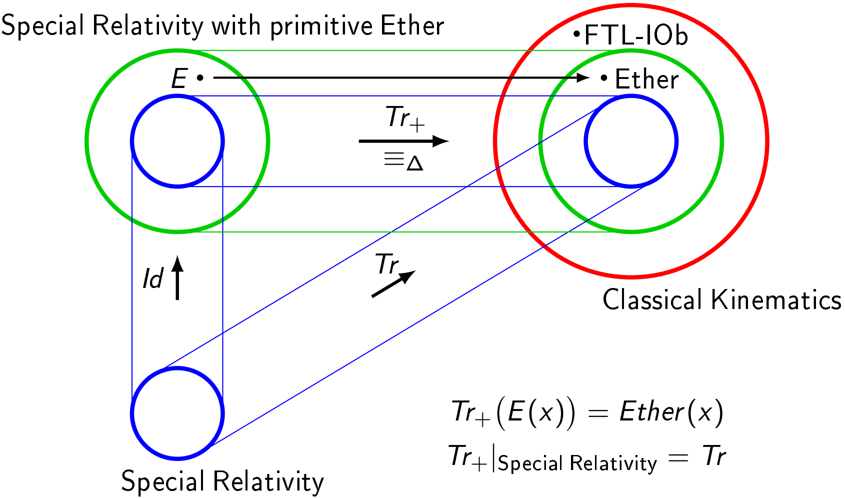


Assuming ***ClassicalKin***$_{\textit{\textbf{Full}}}$, if e is an ether observer and k and h are inertial observers slower than light, then

$$Tr\left(w_{hk}^{SR}\right) = Rad_{\bar{v}_k(e)} \circ w_{hk}^{CK} \circ Rad_{\bar{u}_h(e)}^{-1}$$

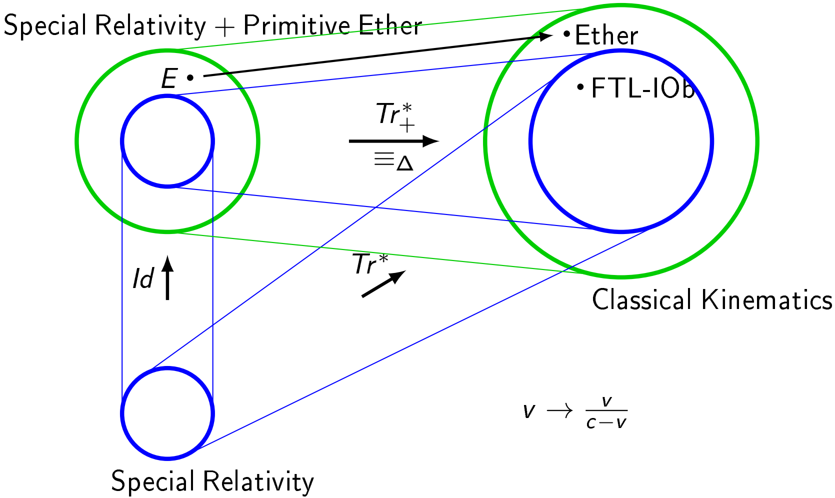
is a Poincaré transformation.

A stronger version of our claims



$$Tr_+(E(x)) = Ether(x)$$

$$Tr_+|_{\text{Special Relativity}} = Tr$$


Special Relativity + Primitive Ether
E •
• Ether
• FTL-IOb
Tr_{+}^{*}
$\equiv_{\Delta}$
Id
Tr^{*}
Classical Kinematics
$v \to \frac{v}{c-v}$
Special Relativity